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CHARGE CORRELATIONS IN HEAVY ION COLLISIONS

A. RAJANTIE

*DAMTP, University of Cambridge
Wilberforce Road, Cambridge CB3 0WA, UK
E-mail: a.k.rajantie@damtp.cam.ac.uk*

When hot quark gluon plasma expands and cools down after an heavy ion collision, charge conservation leads to non-trivial correlations between the charge densities at different rapidities. If these correlations can be measured, they will provide information about dynamical properties of quark gluon plasma.

1. Introduction

The view to the quark gluon plasma phase (QGP) in heavy ion collisions is obscured by events taking place later on. One way around this is to focus on conserved charges, such as electric charge, baryon number or strangeness.^{1,2} Later evolution can only change these charges locally, through pair creation or annihilation, but charge fluctuations with long enough wavelengths will remain unchanged.

One possibility is to consider the fluctuation $\langle \Delta Q^2 \rangle$ of the charge Q in a given rapidity window $\Delta\eta$.^{1,2} Because of charge conservation, this quantity can only change when charges move in and out of the window, but if the window is wide enough, this effect should be small. Therefore, the measured charge fluctuation should reflect the initial value and be different for QGP and hadronic phases.

Ignoring long-range Coulomb forces, one can estimate that in the QGP phase, where the electric charges of the quarks are either $2/3$ or $-1/3$, the fluctuation is $\langle \Delta Q^2 \rangle \approx 0.2N_{\text{ch}}$. In the hadronic phase, elementary charges are ± 1 , and as a consequence the fluctuation is much larger $\langle \Delta Q^2 \rangle \approx 0.7N_{\text{ch}}$.² In principle, this difference could be used to find out whether QGP formed in the early stages of the collision. However, current experimental results are consistent with the hadronic value.^{3,4,5} They are also very close to $\langle \Delta Q^2 \rangle = N_{\text{ch}}$, which would correspond to a purely random distribution

of ± 1 charges.

In this talk, I will further explore the information obtainable from charge fluctuations. In particular, I will consider correlations between charge densities at different rapidity values, and show that they carry quantitative information about different stages of the collision. It is hopefully easier to subtract the contributions due to phenomena taking place in the later stages of the collision, such as decays of hadronic resonances, from these correlations than from the charge fluctuation signal $\langle \Delta Q^2 \rangle$.

2. Diffusion in an expanding system

In the first stages of a heavy ion collision, the expansion in the direction of the beam is much faster than that in the orthogonal directions. The effects I will be discussing are all due to this expansion, and therefore I will not consider the orthogonal directions. I will also assume that the two nuclei are moving at the speed of light so that the collision event is boost invariant. In that case, it is convenient to use the Bjorken coordinates τ and η defined by $t = \tau \cosh \eta$, $z = \tau \sinh \eta$. In these coordinates, the Minkowski metric becomes $ds^2 = d\tau^2 - \tau^{-2} dz^2$, which is the metric of a 1+1 dimensional FRW universe with scale factor $a(\tau) = 1/\tau$. This means that very similar considerations apply to charge considerations in the early universe, as well.

I will assume that the evolution of the charge density is purely diffusive. Charge annihilations in that case have been studied for a long time,⁶ but often without coupling the system to a heat bath. In the presence of a thermal bath, the diffusion equation in Bjorken coordinates for the comoving charge density $\tilde{\rho} = dQ/d\eta$ is

$$\partial_\tau \tilde{\rho} = \frac{D(\tau)}{\tau^2} \partial_\eta^2 \tilde{\rho} + \partial_\eta \xi_\eta, \quad (1)$$

where ξ_η is a stochastic variable that describes thermal noise. It has a symmetric Gaussian distribution with a two-point function

$$\langle \xi_\eta(\tau, \eta) \xi_\eta(\tau', \eta') \rangle = 2D(\tau) G_{\text{eq}}(\tau) \delta(\tau - \tau') \delta(\eta - \eta'). \quad (2)$$

The amplitude of the noise is given by $G_{\text{eq}}(\tau)$, which depends on the temperature and can be written as $G_{\text{eq}}(\tau) = q^2 \tau n_{\text{eq}}(\tau)$, where q is the elementary charge and $n_{\text{eq}}(\tau)$ is the equilibrium particle density.

The stochastic term in Eq. (1) can be eliminated by considering the two-point function $G(\tau, \eta - \eta') = \langle \tilde{\rho}(\tau, \eta) \tilde{\rho}(\tau, \eta') \rangle$. It satisfies the equation of motion

$$\partial_\tau G(\eta) = \frac{2D(\tau)}{\tau^2} \partial_\eta^2 [G(\eta) - G_{\text{eq}}(\tau) \delta(\eta)]. \quad (3)$$

The Fourier modes $G(k_\eta) = \int d\eta e^{ik_\eta \eta} G(\eta)$ satisfy

$$\partial_\tau G(k_\eta) = -\frac{2D(\tau)k_\eta^2}{\tau^2} [G(k_\eta) - G_{\text{eq}}(\tau)]. \quad (4)$$

If we assume that initially at $\tau = \tau_{\text{ini}}$, the system is in equilibrium, i.e., $G(\tau_{\text{ini}}) = G_{\text{eq}}(\tau_{\text{ini}})$, then the solution is

$$G(\tau, k_\eta) = G_{\text{eq}}(\tau) - \int_{\tau_{\text{ini}}}^{\tau} d\tau' e^{-\frac{1}{2}\Delta^2(\tau')k_\eta^2} \dot{G}_{\text{eq}}(\tau'), \quad (5)$$

where $\Delta^2(\tau') = 4 \int_{\tau'}^{\tau} d\hat{\tau} [D(\hat{\tau})/\hat{\tau}^2]$.

Back in coordinate space, we have

$$G(\tau, \eta) = G_{\text{eq}}(\tau)\delta(\eta) - \int_{\tau_{\text{ini}}}^{\tau} d\tau' \frac{e^{-\eta^2/2\Delta^2(\tau')}}{\sqrt{2\pi\Delta^2(\tau')}} \dot{G}_{\text{eq}}(\tau') \quad (6)$$

Assuming that the charged particles are ultrarelativistic, their equilibrium density depends on the temperature as $n_{\text{eq}} \propto T^3$. Entropy density scales in the same way, and therefore the temperature decreases with the expansion as $T \propto \tau^{-1/3}$, meaning that G_{eq} remains constant. This means that to a good approximation, \dot{G}_{eq} is simply a sum of delta functions at phase transitions and other non-adiabatic events. Consequently, $G(\tau, \eta)$ becomes a sum of Gaussians with different heights and widths.

As an example, let us assume that the system is initially in thermal equilibrium in the QGP phase so that $G_{\text{eq}}(\tau_{\text{ini}}) = G_{\text{QGP}}$. When the system enters the hadronic phase at $\tau = \tau_{\text{tr}}$, G_{eq} jumps discontinuously to G_{had} , which is higher because the elementary charge q is ± 1 instead of $2/3$ or $-1/3$. Thus, $\dot{G}_{\text{eq}}(\tau) = (G_{\text{had}} - G_{\text{QGP}})\delta(\tau - \tau_{\text{tr}})$, and the final two-point function is

$$G(\tau, \eta) = G_{\text{had}}\delta(\eta) - (G_{\text{had}} - G_{\text{QGP}}) \frac{e^{-\eta^2/2\Delta^2(\tau_{\text{tr}})}}{\sqrt{2\pi\Delta^2(\tau_{\text{tr}})}}. \quad (7)$$

The charge fluctuation signal $\langle \Delta Q^2 \rangle$ can be written in terms of $G(\tau, \eta)$ as

$$\langle \Delta Q^2 \rangle = \frac{1}{\Delta\eta^2} \int_{-\Delta\eta/2}^{\Delta\eta/2} d\eta d\eta' G(\tau, \eta - \eta'). \quad (8)$$

It is easy to see that it, indeed, gives the QGP result if $\Delta\eta \gg \Delta(\tau_{\text{tr}})$ and the hadronic result otherwise.

If we then assume that the hadrons become non-relativistic and start to annihilate at a later time τ_{nr} , the delta function peak spreads into a Gaussian

$$G(\tau, \eta) = G_{\text{had}} \frac{e^{-\eta^2/2\Delta^2(\tau_{\text{nr}})}}{\sqrt{2\pi\Delta^2(\tau_{\text{nr}})}} - (G_{\text{had}} - G_{\text{QGP}}) \frac{e^{-\eta^2/2\Delta^2(\tau_{\text{tr}})}}{\sqrt{2\pi\Delta^2(\tau_{\text{tr}})}}. \quad (9)$$

It is interesting to note that at short enough distances ($\eta \lesssim \Delta(\tau_{\text{nr}})$), the correlator is positive.^{6,7} This also means that for small $\Delta\eta$, the charge fluctuation should behave as $\langle \Delta Q^2 \rangle \propto \Delta\eta^2$.

3. Long-range forces

The discussion in Section 2 applies to global charges such as baryon number or strangeness, but for electric charges, one has to take into account the long-range Coulomb interaction, and the motion of the charges is not purely diffusive. By combining Ohm's law $\vec{j} = \sigma \vec{E}$ with Gauss's law $\vec{\nabla} \cdot \vec{E} = \rho$, the diffusion equation gets modified and becomes

$$\partial_\tau \tilde{\rho} = \frac{D(\tau)}{\tau^2} \partial_\eta^2 \tilde{\rho} - \sigma(\tau) \tilde{\rho} + \partial_\eta \xi_\eta. \quad (10)$$

It is instructive to note that when all the coefficients are time-independent, one finds that the equilibrium two-point function is

$$G(k) = \frac{G_{\text{eq}} k^2}{k^2 + \sigma/D}, \quad (11)$$

which shows that the Debye screening mass m_D is given by $m_D^2 = \sigma/D$.

Analogously to Eq. (3), we can derive an equation of motion for the two-point function

$$\partial_\tau G(k_\eta) = -\frac{2D(\tau)k_\eta^2}{\tau^2} [G(k_\eta) - G_{\text{eq}}(\tau)] - 2\sigma(\tau)G(k_\eta). \quad (12)$$

Assuming that initially $G(k_\eta)$ vanishes, the solution is

$$G(\tau, k_\eta) = G_{\text{eq}}(\tau) - \int_0^\tau d\tau' e^{-\frac{1}{2}\Delta^2(\tau')k_\eta^2 - 2\Sigma(\tau')} \left[2\sigma(\tau')G_{\text{eq}}(\tau') + \dot{G}_{\text{eq}}(\tau') \right], \quad (13)$$

where $\Sigma(\tau') = \int_{\tau'}^\tau d\hat{\tau} \sigma(\hat{\tau})$. Again, this is simply a superposition of Gaussians, and can therefore be easily Fourier transformed back to coordinate space.

As an example, let us consider a simple case in which G_{eq} jumps from 0 to 1 at $\tau_{\text{tr}} = 1$. Furthermore, we assume that $D(\tau) = \beta\tau$, where β is constant, and $\sigma(\tau) = m_D^2 D(\tau)$ with constant m_D^2 . The correlator $G(\tau, k_\eta)$ has been plotted in the left panel of Fig. 1 for $m_D^2 = 0$ [global charge, from Eq. (7)] and for $m_D^2 = 0.01$ [local charge with long-range forces]. One can see that for a local charge, the correlator is more strongly peaked around zero.

We then imagine that G_{eq} drops instantaneously to zero at $\tau_{\text{nr}} = 10$ as the charged particles become non-relativistic. The correlators for global

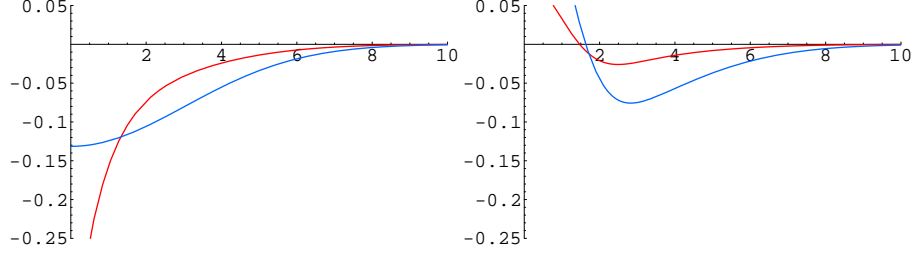


Figure 1. $G(\tau, k_\eta)$ as a function of k_η at $\tau = 10$ (left) and at $\tau = 13$ (right). The two curves correspond to $m_D^2 = 0$ and $m_D^2 = 0.01$.

and local charges at $\tau = 13$ are shown in the right panel of Fig. 1. Because of the long-range forces, annihilation is faster and

4. Conclusions

We have seen that the charge correlators measured at late times carry detailed quantitative information about the properties of the system at different stages of the collision. Particle-antiparticle pairs produced later on by neutral resonances etc. will add an extra contribution to these correlations, but if it is properly understood it can be subtracted, at least in principle.

The signals will be much stronger in global charges, i.e., baryon number and strangeness than in the electric charge, but unfortunately they are also much more difficult to measure. It is possible, though, that some of the interesting features survive even in the electric charge correlators, but a more detailed calculation is needed to find out if that is actually the case.

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